

Hierarchical basis functions and efficient multilevel solvers

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Abstract

We present a hierarchical basis for incomplete $\mathbf{H}(\text{curl})$ -conforming finite element spaces on tetrahedral meshes. A certain amount of orthogonality between basis functions of different orders is obtained through the requirement that the Nédélec interpolation of higher order basis functions vanishes in lower order finite element spaces. We also present two versions of a multiplicative Schwarz preconditioner, which are used together with the conjugate gradient method to efficiently solve the linear system that results from the finite element discretization of the vector wave equation. Similar preconditioners have previously been proven to be very efficient in schemes with second order elements, but here we focus on the extension to schemes with elements of third and fourth order. Numerical experiments are used to show the very good performance of the presented schemes. In these experiments we also find that many other bases, which previously have been presented in the literature, give very similar performance when elements of third order are used. However, when fourth order elements are used, the basis we present here leads to clearly better performance.