modelling printed circuit antennas by boundary method

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1 Introduction

Printed circuits are made of very thin metallic opened shells put on a dielectric material. It is an essential problem to determine their response when they are immersed into an electromagnetic field.
When the field is at low frequency and when the metallic shell is closed (it entirely covers the dielectric), the problem has been solved by introducing potentials; but those methods do not permit to treat high frequencies and cannot take into account opened shells. We here present a formulation suited to this problem.

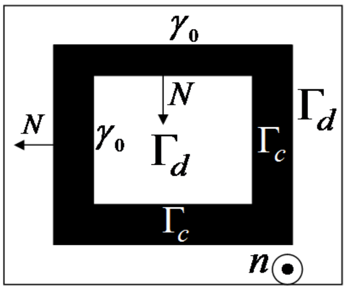

Figure 1: The Annulus circuit

2 Physical Problem

The system is made of a dielectric plate Ω of permittivity ($\varepsilon = \varepsilon_0 \varepsilon_r$) and permeability μ_0; on its surface Γ is a conducting circuit of very small thickness d and conductivity σ. The outside domain is the air. This system is put into a high frequency electromagnetic source field (e^s, h^s) at frequency ω and the penetration-depth δ is such that: $\delta \gg d$; so, in the circuit, electric field e and current density $j = \sigma e$ are constant over d. Moreover, as d is greatly smaller than the other dimensions we consider that the metallic circuit Γ_c is geometrically a part of Γ; the rest of Γ is called Γ_d (dielectric part) and thus $\Gamma = \Gamma_c \cup \Gamma_d$. The line between Γ_c and Γ_d is called γ_0. n is the outside normal to Γ and N is the outside normal to γ_0 on Γ. The example of an annulus circuit is showed in Fig. 1.
We define surface current J_c by:

$$\begin{array}{lll} J_c = & \sigma\, d\, e & \text{on } \Gamma_c \\ J_c = & 0 & \text{on } \Gamma_d \end{array} \qquad (1)$$

According to Maxwell's equation, the jumps across Γ of the tangential fields are

$$[n \times h]_\Gamma = J_c, \quad [n \times e]_\Gamma = 0.$$

3 Variational Formulation

In the case of a dielectric material completely covered with a conducting thin shell we have proposed [1] a variational formulation to compute the fields and the current in the shell which uses the integral representation of the electromagnetic fields at high frequencies obtained by Stratton-Chu's formulae [2], [3] but, in the case where the dielectric is partially covered, it cannot be applied because the lineic charges appearing on γ_0 need to be taken into account.
Here, we also use Stratton-Chu's formulae.
We call e^a, h^a the tangential components of the electromagnetic fields by the side of the air and G_a the Green function in the air:

$$G_a(x,y) = \frac{e^{ik_a|x-y|}}{4\pi|x-y|} \text{ with } \mathrm{k}_\mathrm{a}^2 = \varepsilon_0 \mu_0 \omega^2.$$

We have e^d and h^d by the side of Ω with Green fonction G_d in the dielectric:

$$G_d(x,y) = \frac{e^{ik_d|x-y|}}{4\pi|x-y|} \text{ with } \mathrm{k}_\mathrm{d}^2 = \varepsilon_0 \varepsilon_\mathrm{r} \mu_0 \omega^2.$$

We define the unknown quantities e and h:

$$n \times h = n \times h^a = n \times h^d + J_c, \qquad (2)$$

$$n \times e = n \times e^a = n \times e^d. \qquad (3)$$

These equations are very important because they permit, after using boundary methods, to present a variational formulation implying only Γ by taking as unknown quantities the tangential components of fields e and h and the current J_c on Γ. We use the integral representation of the electromagnetic fields in the air and in the dielectric and write them under a variational form with test functions on Γ. We obtain:

$$-i\omega\varepsilon_0 \int_\Gamma\int_\Gamma (n(x)\times e'(x))\cdot(n(y)\times e(y))\,(G_a+\varepsilon_r\,G_d)$$
$$-\frac{1}{i\omega\mu_0}\int_\Gamma\int_\Gamma \mathrm{div}_\Gamma(n(x)\times e'(x))$$
$$\mathrm{div}_\Gamma(n(y)\times e(y))\,(G_a+G_d)$$
$$+\int_\Gamma\int_\Gamma (n(x)\times(e'(x))\times(n(y)\times h(y))\cdot\mathrm{grad}_x(G_a+G_d)\cdot$$
$$-\int_\Gamma\int_{\Gamma_c}((n(x)\times e'(x))\times J_c)\cdot\mathrm{grad}_x G_d$$
$$+\frac{1}{2}\int_{\Gamma_c} e'\cdot J_c = -\int_\Gamma (n\times e')\cdot h^s, \qquad (4)$$

and:

$$i\omega\mu_0\int_\Gamma\int_\Gamma (n(x)\times(h'(x))\cdot(n(y)\times h(y))\,(G_a+G_d)$$
$$-i\omega\mu_0\int_\Gamma\int_{\Gamma_c}(n(x)\times h'(x))\cdot J_c(y)\,G_d$$
$$+\frac{1}{i\omega\varepsilon_0}\int_\Gamma\int_\Gamma \mathrm{div}_\Gamma(n(x)\times h'(x))$$
$$\mathrm{div}_\Gamma(n(y)\times h(y))(G_a+\frac{G_d}{\varepsilon_r})$$
$$+\frac{1}{i\omega\varepsilon_0}\int_\Gamma\int_{\gamma_0}\mathrm{div}_\Gamma(n(x)\times h'(x))\;J_c\cdot N\,\frac{G_d}{\varepsilon_r}$$
$$+\int_\Gamma\int_\Gamma (n(x)\times h'(x))\times(n(y)\times e(y))\cdot\mathrm{grad}_x(G_a+G_d)$$
$$= -\int_\Gamma (n\times h')\cdot e^s. \qquad (5)$$

We notice that we can not replace J_c by its value (1) and limit the problem to two unknown quantities because the problem will then be badly defined.
The double integral $\int_\Gamma\int_{\gamma_0}$, where $\int_{\gamma_0}$ is a line integral, permits to take into account the lineic charges which appear on γ_0. It comes from $\mathrm{div}_\Gamma J_c$ which is null except on γ_0 where it is equal to $-J_c\cdot N\,\delta_{\gamma_0}$.
From (1) with J' as a test function defined only on Γ_c, we write :

$$\int_{\Gamma_c} J_\Gamma\cdot J' - \int_{\Gamma_c}\sigma d\,e\cdot J' = 0\,, \qquad (6)$$

Equation (6) is very important because it permits to have the unicity of the solution of the formulation.
It appears that the formulation, made of (4) (5) and (6), depends only on product σd and not directly on σ or d.
For the discretization, surface Γ is meshed into triangles. Fields e and h, which appear by their tangential components, are discretized with Nédélec's edge elements and the degrees of freedom are the fields circulations along the edges. Current J_c is discretized with edge elements rotated by $\pi/2$ the degrees of freedom being the fluxes through the edges.

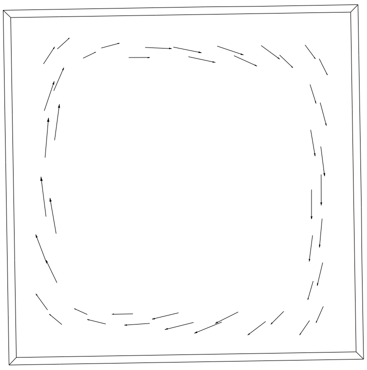

Figure 2: The current density in a conducting annulus with a slit

4 Numerical Results

The presented variational formulation, which only uses the unknown quantities in the air on the surface of the domain, has been applied to many examples. All the obtained results show that surface current is well presented in a printed circuit or generally in an opened shell as soon as the thickness of the metal is very small.
Fig 2 shows surface current J_c in the case of a dielectric plate ($L = l = 12\,cm$, $H = 2\,cm$, $\varepsilon_r = 25$), on which there is a conducting annulus (width $= 1$ cm, $\sigma d = 0.01\ S$) with a slit in the middle of the left side; it is put into a plane wave.
This formulation can be used, in particular, to model the antennas used in MRI.

References

[1] B. Kebaili, B. Bandelier and F. Rioux-Damidau "Modelling a thin shell submitted to an electromagnetic wave" *IEEE Trans. Magn.*, vol 37, 2001, pp. 3277-3280.

[2] J. A. Stratton, *Electromagnetic theory,* Mac Graw Hill, New York, 1941.

[3] V. Levillain, "Couplage éléments finis-équations intégrales pour la résolution des équations de Maxwell en milieu hétérogène," *PHD Thesis,* Polytechnique, Palaiseau, 1991.