

# Metamaterials, a challenge for homogenization theory

Alain Bossavit

LGEP, CNRS, 11 Rue Joliot-Curie, 91192 Gif-sur-Yvette, France

Bossavit@lgep.supelec.fr

Metamaterials are crystal-like composites, where high-frequency electromagnetic fields propagate in surprising ways. To the difference of what happens in standard homogenization theory, effective constants for the equivalent homogeneous medium, in the Full Maxwell case, depend on the frequency, and may have a negative real part in a specific range of frequencies. This results in emergent properties: refractive lenses with only plane interfaces, concave lenses which refocus divergent beams into tightly parallel ones, prisms that "bend light the wrong way", etc. Such "negative index materials" (NIM) have been engineered, and are actively studied.

Metamaterials thus challenge standard homogenization theory. The latter posits a so-called "cell problem", set over the generative cell of the crystal, with appropriate pseudo-periodic boundary conditions, from which effective homogenized coefficients  $\mu_{\text{eff}}$  and  $\varepsilon_{\text{eff}}$  are computed. These are averages (a kind of mix of arithmetic and harmonic mean) of the physical, spatially periodic  $\varepsilon$  and  $\mu$ , which are essentially positive, so one fails to comprehend how negative homogenized coefficients could result from such a procedure.

Learning from the physicists give the essential cue in this respect: the NIM behavior is caused by "internal resonance" phenomena, due to deliberate arrangement of materials inside the cell in order to create the equivalent of RLC circuits. Such resonance phenomena are acknowledged in the version of "frequency-dependent homogenization" we propose, in which the cell-problem's differential operator is akin to  $\omega^2 + \text{curl} \circ \text{curl}$ , instead of the  $-\text{div} \circ \text{grad}$  of standard theory, so that computed effective coefficients do depend on frequency. Energetic considerations, based on the variational formulation of the problem, then easily show that, indeed, effective coefficients, complex-valued, can cross the imaginary axis at definite frequencies, thus acquiring negative real parts.

Solving this cell problem is an alternative to the now popular "Bloch–Floquet waves" method (which can be understood, actually, as solving the cell problem by spatial Fourier analysis). It can be done, as we'll show, by using edge- and face-based variables, in the spirit of the modern "generalized finite difference" approach. Hence a tool to design and optimize metamaterials.