Geometrical localization of the degrees of freedom for Whitney elements of higher order

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Whitney elements on simplices are perhaps the most widely used finite elements in computational electromagnetics. Many implementations using Whitney elements of the lowest polynomial order $k = 1$ exist. This is not the case for higher orders $k > 1$, due to the complexity in generating element basis functions and localizing the corresponding degrees of freedom on the mesh tetrahedra.

In this paper we wish to give a definition of Whitney forms of polynomial degree higher than one on simplices together with a geometrical localization of the degrees of freedom associated to these forms. We provide a conveniently implementable basis for these elements on simplicial meshes. As for Whitney forms of degree one, the basis is expressed only in terms of the barycentric coordinates of the simplex.

The key points in this construction are two: (1) these forms should satisfy a partition of unity property, (ii) being a larger number with respect to those of degree $k = 1$ they are associated to a finer partition in each tetrahedron.

Very first numerical tests, in the case of edge elements, show that the convergence rate of the proposed method is in agreement with the edge element theory, i.e., the approximation error behaves as $O(h^k)$, where h is the maximal diameter of the mesh tetrahedra.

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